Finite Element Analysis of Elastomers

Nordic Rubber Conference

Malmö 9-10/4 -14

Ass. prof.
Per-Erik Austrell
Lund University
o **Ass. Professor** (Univ. lektor) at the Structural Mechanics dep. at Lund University

o **Lecturer** in a basic course (150 stud.) in Mechanics (Newton's laws) and in an advanced course (30 stud.) in Structural Dynamics

o **Specialist** in modelling Rubber Units; material modelling and Finite Element Analysis, interest in applications, cooperation with companies

o Also windsurfer, golfer, and inline skater ... when possible
DISPOSITION

- Background & some projects
- Modelling rubber material properties
- FEA of rubber components
Material modelling aiming at FEA. Experimental and theoretical work.

Cont. into FEA procedures: Overlay method etc.

MODELING OF ELASTICITY AND DAMPING FOR FILLED ELASTOMERS

FINITE ELEMENT PROCEDURES IN MODELLING THE DYNAMIC PROPERTIES OF RUBBER

ANDERS K. OLSSON

Structural Mechanics

Doctoral Thesis
DEVELOPMENT OF AN ELASTOMERIC FINGER JOINT

Work with P-I Brånemark -99-01
The knuckle joint was replaced
Focus on choice of material and on shape optimization by FE-analysis
The existing phrostesis was re-designed for better handling of compressive forces
RUBBER COVERED ROLLERS

Extensive work going on for about 2-3 years.
Documentation about 230 pages
A stand alone software was developed for roller simulations etc.

MODELLERING AV KONTAKTEN MELLAN PRESSÖR- OCH KYLVALS

Per-Erik Austrell
Avdelningen för byggnadsmekanik
Lunds Tekniska Högskola
14 februari 2003

Medarbetare LTH: Anders K Olsson, Lars Andersson, Paul Håkansson, Anders Melander och Ted Bentsson
Hole filling shock absorber by using computer simulations

Excellent fatigue properties

A patent application was sent in.

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UTVECKLING AV ÄNDLÄGESDÄMPARE FÖR BOSCH REXROTH TEKNIK AB

PER-ERIK AUSTRELL, BJÖRN STENBOM, MAGNUS LINDVALL och LARS-OLOF ANDERSSON
DISPOSITION

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SCHEMATIC MATERIAL STRUCTURE OF A CARBON-BLACK-FILLED RUBBER VULCANIZATE

crosslinks
polymer chains
carbon black
Mechanical analogy qualitatively consistent with microstructure and mechanical properties:

\[ \sigma = \sigma_e + \sigma_v + \sigma_f \]

including elastic, viscous, and frictional properties.
Hysteresis loops at slow harmonic loading:

- Deviations from pure viscoelastic behavior:
  - The losses remain at slow loading rate.
  - The hysteresis loop has sharp edges.
  - The dynamic modulus depends on amplitude.

Quasi-static test (0.05 Hz) harmonic loading.
MATERIAL MODELS FOR RUBBER IN FEA

Nonlinear elastic:
* Hyperelastic models
  Strain energy based Polynomial or Ogden formulation

Damping included:
* Visco-hyperelastic models with purely rate dependent damping
* Viscoplastic models that also includes rate independence (research codes)

Damage also included:
Research codes capable of modeling Mullin’s effect
HYPERELASTIC MATERIAL MODELS

**neo-Hooke model:**
One parameter; \( C_{10} \)

**Yeoh model:**
Three parameters;
\( C_{10}, C_{20} \) & \( C_{30} \)

\[ C_{10} = G/2 \] with \( G; \) initial shear modulus

Comparison in compression/tension and in simple shear of Neo-Hooke (dashed line) and Yeoh (solid line) models.

Properties of the Yeoh model:
- It gives a good fit to experiments on carbon-black-filled rubbers.
- Only three parameters have to be determined.
- It is possible to obtain the parameters from a shear test only.
- Fitting usually gives stable models
LARGE STRAIN VISCOELASTICITY

Two options in commercial codes:

a) Small harmonic vibrations on a large static (hyper)elastic deformation

b) Large strain transient analysis
Sinusoidal test in simple shear:

Dynamic modulus: \( G_{dyn} = \tau_0 / \kappa_0 \)

Equivalent phase angle: \( \delta_e = U_c / (\pi \tau_0 \kappa_0) \)

\( U_c \): hysteresis work (viscoelastic \( U_c = \pi \tau_0 \kappa_0 \sin \delta \))

Rem. Linear viscoelastic:
- Elliptic hysteresis curve
- Dynamic modulus dependent on frequency only
Experimental results for 4 rubbers with increasing amount of filler in harmonic loading at 1 Hz:

**dynamic modulus**

**phase angle**

Amplitude dependence

The dynamic modulus $G_{dyn}$ declines with increasing amplitude.
ONE-DIMENSIONAL MODELS

- Viscoelastic models:

  Frequency dependence of the dynamic modulus:

  1) low, 2) medium and 3) high frequency

  no amplitude dependence

- Elastic plastic models:

  Amplitude dependence of the dynamic modulus:

  no frequency dependence

Combining yields a 5-parameter model ....
ONE-DIMENSIONAL ... cont.

.... giving both amplitude and frequency dependence:

Dynamic modulus

Phase angle
A GENERALIZED VISCO-FRICTIONAL MODEL

A model with separable amplitude and frequency dependence consistent with experiments and microstructure:

\[ \sigma = \sigma_e + \sigma_v + \sigma_f \]

- Elastic part
- Viscous part
- Frictional part

Viscous part $\Rightarrow$
Frequency dependence
Frictional part $\Rightarrow$
Amplitude dependence
EXAMPLE FIT; HNBR - Generalized model

Each color represents a specific shear strain amplitude:
red; 1% blue; 3% green; 7% blue-green; 12%
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FE-CODES FOR NON-LINEAR ANALYSIS OF RUBBER UNITS

Requirements:
The codes must handle:

- Large deformations and strains
- Almost incompressible materials
- Contact conditions
- Hyperelastic, viscoelastic and possibly also elastoplastic material models
Useful for:

- Visualizing deformations in rubber units
- Generating spring characteristics
- Computing principal stresses for judgement of crack propagation risk
- etc
OVERVIEW OF THE Overlay METHOD

Going from testing to FE-model:
- Harmonic shear test
- Viscoplastic model (1D)
- Mesh assembly with several layers on the same nodes
- FE-model

The resulting FE-model takes into account:
Non-linear elasticity, rate dependence, and amplitude dependence
EXAMPLE: RADially LOADED BUSHING

Verification example: Harmonically loaded NR bushing

Double shear specimens

... and cylindric bushings were manufactured

... by the same 50 Shore NR-material

A FE-model based on the overlay method was compared to measured data for the bushing at various frequencies and amplitudes
EXAMPLE: RADIAALLY LOADED ... cont. 1

**Shear test vs. 1D-model:**
EXAMPLE: RADIALY LOADED ... cont. 2

Rem. Taking thermal stresses from the manufacturing into account lowers the stiffness of the model.
Contact:
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Thank you for listening!
Two NUTEK projects on rubber modelling: 93-99

Thesis -97: *Modeling of elasticity and damping for filled elastomers*

Bio-mechanics; Elastomeric finger joint - P-I Brånemark

Roller modelling and simulation - Tetra Pak (ongoing)

Rubber bushing modelling - Volvo Cars (ongoing)

Modelling of gaskets in plate heat exchangers - Alfa Laval

Development of a shock absorber - Bosch-Rexroth

Supervisor for about 20 Masters Thesis works on rubber components

Supervisor for PhD student Anders K Olsson (tekn. dr.) and other PhD students

Leif Kari KTH and P-E A organizers for the European Conference on Constitutive Modelling of Rubber -05 (www.eccmr.org)


- 2001 Austrell P-E. et. al., *A method to analyze the dynamic behavior of carbonblack filled rubber components using standard FE-codes*, proceedings of the second European conference on constitutive models for rubber (ECCMR), Hannover, Germany, September 2001

LITERATURE ... Cont. 1


2012 Austrell & Olsson, *Modelling procedures and properties of rubber in rolling contact*, Polymer Testing No 32.


**DYNAMIC PROPERTIES**

Left: Dyn. Modulus  
Right: Loss angle
FITTING THE GENERALIZED MODEL

Goal function:

\[ \psi = (1 - \alpha) \sum_{i=1}^{m} \left( \frac{d_{\text{dyn},i} - d_{\text{exp},i}}{d_{\text{exp},i}} \right)^2 + \alpha \sum_{i=1}^{m} \left( \frac{G_{\text{dyn},i} - G_{\text{exp},i}}{G_{\text{exp},i}} \right)^2 \]

... model parameters (left) are fitted to the double shear test by minimizing the goal function.

the weight factor \( \alpha \) gives the importance of dynamic modulus vs damping.
FIT OF THE GENERALIZED MODEL... cont.

... software in MATLAB

... choose the number of viscous and frictional elements

Also the weight damping vs modulus
EXAMPLE FIT; HYPALON

small amplitude dependence and big frequency dependence
EXAMPLE FIT; SILICON

big amplitude dependence and small frequency dependence
**CHOICE OF ANALYSIS TYPE**

**Standard codes:**
Uses stiffness and mass matrices and tangential directions and (dynamic) equilibrium iterations.

**Explicit codes:**
Dynamic solution method with direct time stepping no equilibrium iterations. Static loading can be simulated dynamically.

- Standard codes gives correct solutions when convergent, not always true for explicit codes.
- Contact works better in Explicit codes.
- Complete incompressibility is not possible in explicit codes*

Rem. *) But B = 100 G is OK.
CHOICE OF ANALYSIS TYPE ... cont.

Explicit or implicit analysis?

Example 3D-Analysis:
Tetra Pak package sealing

Memory/CPU time:

Abaqus-Standard:
2.5 GB / 250 CPU hours

Abaqus-Explicit:
80MB / 7 CPU hours
Material modelling of rubber aiming at finite element analysis.
Both experimental and theoretical work.
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